

Wormhole Solutions in Gauss-Bonnet-Born-Infeld Gravity

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A new class of solutions which yields an $(n+1)$ -dimensional spacetime with a longitudinal nonlinear magnetic field is introduced. These spacetimes have no curvature singularity and no horizon, and the magnetic field is non singular in the whole spacetime. They may be interpreted as traversable wormholes which could be supported by matter not violating the weak energy conditions. We generalize this class of solutions to the case of rotating solutions and show that the rotating wormhole solutions have a net electric charge which is proportional to the magnitude of the rotation parameter, while the static wormhole has no net electric charge. Finally, we use the counterterm method and compute the conserved quantities of these spacetimes.

I. INTRODUCTION

The intriguing possibility that our Universe is only a part of a higher dimensional spacetime has raised a lot of interest in physics community recently. In string theory extra dimensions are necessary since superstring theory requires a ten-dimensional spacetime to be consistent from the quantum point of view. The effect of string theory on the left hand side of field equations of gravity is usually investigated by means of a low energy effective action which describes gravity at the classical level [1]. This effective action consists of the Einstein-Hilbert action plus curvature-squared terms and higher powers as well, and in general give rise to fourth order field equations and bring in ghosts. However, if the effective action contains the higher powers of curvature in particular combinations, then only second order field equations are produced and consequently no ghosts arise [2]. The effective action obtained by this argument is precisely of the form proposed by Lovelock [3]. On the other hand, the dynamics of D-branes and some soliton solutions of supergravity is governed by the Born-Infeld action [4], and therefore if one want to couple electromagnetic field with gravity it is more suitable to put the energy momentum of Born-Infeld electromagnetic field

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on the right hand side of the field equations. While the Lovelock gravity was proposed to have field equations with at most second order derivatives of the metric [3], the nonlinear electrodynamics proposed, by Born and Infeld, with the aim of obtaining a finite value for the self-energy of a point-like charge [5]. The Lovelock gravity reduces to Einstein gravity in four dimensions and also in the weak field limit, while the Lagrangian of the Born-Infeld (BI) electrodynamics reduces to the Maxwell Lagrangian in the weak field limit. Considering the analogy between the Lovelock and the Born-Infeld terms, which are on similar footing with regard to string corrections on the gravity side and electrodynamic side, respectively, it is plausible to include both these corrections simultaneously. Here we restrict ourself to the first three terms of Lovelock gravity. The first two terms are the Einstein-Hilbert term with cosmological constant, while the third term is known as the Gauss-Bonnet term. In recent decades, the exact solutions of Einstein-Gauss-Bonnet gravity and their properties were studied. For example, static spherically symmetric black hole solutions of the Gauss-Bonnet gravity were found in Ref. [6]. Black hole solutions with nontrivial topology in this theory were also studied in Refs. [7, 8, 9]. NUT charged black hole solutions of Gauss-Bonnet gravity and Gauss-Bonnet-Maxwell gravity were obtained in [10]. Also rotating black hole solutions of Gauss-Bonnet-Maxwell and Born-Infeld-Gauss-Bonnet gravity have been studied in [11, 12].

Currently, there exist some activities in the field of wormhole physics following, particularly, the seminal works of Morris, Thorne and Yurtsever [13]. Morris and Thorne assumed that their traversable wormholes were time independent, non-rotating, and spherically symmetric bridges between two universes. The manifold of interest is thus a static spherically symmetric spacetime possessing two asymptotically flat regions. These kinds of wormholes could be threaded both by quantum and classical matter fields that violate certain energy conditions at least at the throat known as exotic matter. On general grounds, it has recently been shown that the amount of exotic matter needed at the wormhole throat can be made arbitrarily small thereby facilitating an easier construction of wormholes [14]. Lorentzian wormholes in spacetimes with more than four dimensions were analyzed by several authors [15]. In particular, wormholes in Gauss-Bonnet gravity were considered in Ref. [16]. In this paper we are looking for the $(n + 1)$ -dimensional horizonless solutions in Born-Infeld-Gauss-Bonnet gravity. The motivation for constructing these kinds of solutions is that they may be interpreted as wormhole. Here, we will find these kinds of horizonless solutions in the Born-Infeld-Gauss-Bonnet gravity, and use the counterterm method to compute the conserved quantities of the system. The outline of our paper is as follows. In Sec. II we give a brief review of the field equations of Born-Infeld-Gauss-Bonnet gravity, present a new class of static wormhole solutions which produces longitudinal magnetic field, and then we

consider the properties of the solutions as well as the weak energy condition. In Sec. III we endow these spacetime with global rotations and apply the counterterm method to compute the conserved quantities of these solutions. Finally, we finish our paper with some closing remarks.

II. STATIC WORMHOLE SOLUTIONS

We begin this section with a brief review of Einstein-Gauss-Bonnet gravity in the presence of nonlinear Born-Infeld electromagnetic field. The field equations of this theory in the presence of a negative cosmological constant may be written as

$$G_{\mu\nu} - \frac{n(n-1)}{2l^2}g_{\mu\nu} + \alpha H_{\mu\nu} = \frac{1}{2}g_{\mu\nu}L(F) + \frac{2F_{\mu\lambda}F_{\nu}{}^{\lambda}}{\sqrt{1 + \frac{F^2}{2\beta^2}}}, \quad (1)$$

$$\partial_{\mu} \left(\frac{\sqrt{-g}F^{\mu\nu}}{\sqrt{1 + \frac{F^2}{2\beta^2}}} \right) = 0, \quad (2)$$

where $G_{\mu\nu}$ is the Einstein tensor and $H_{\mu\nu}$ is the divergence-free symmetric tensor

$$H_{\mu\nu} = 2R_{\mu}{}^{\rho\sigma\lambda}R_{\nu\rho\sigma\lambda} - 4R^{\rho\sigma}R_{\mu\rho\nu\sigma} + 2RR_{\mu\nu} - 4R_{\mu\lambda}R^{\lambda}{}_{\nu} - \frac{1}{2}g_{\mu\nu} \left(R_{\kappa\lambda\rho\sigma}R^{\kappa\lambda\rho\sigma} - 4R_{\rho\sigma}R^{\rho\sigma} + R^2 \right). \quad (3)$$

In Eq. (1) β is the Born-Infeld parameter with dimension of mass, $F^2 = F^{\mu\nu}F_{\mu\nu}$ where $F_{\mu\nu}$ is electromagnetic field tensor, and $L(F)$ is the Born-Infeld Lagrangian given as

$$L(F) = 4\beta^2 \left(1 - \sqrt{1 + \frac{F^2}{2\beta^2}} \right). \quad (4)$$

In the limit $\beta \rightarrow \infty$, $L(F)$ reduces to the standard Maxwell form $L(F) = -F^2$, while $L(F) \rightarrow 0$ as $\beta \rightarrow 0$. Equation (1) does not contain the derivative of the curvatures, and therefore the derivatives of the metric higher than two do not appear.

Here, we want to obtain the $(n+1)$ -dimensional solutions of Eqs. (1) and (2) which produce longitudinal magnetic fields in the Euclidean submanifold spans by the x^i coordinates ($i = 1, \dots, n-3$). We assume that the metric has the following form:

$$ds^2 = -\frac{r^2}{l^2}dt^2 + \frac{dr^2}{f(r)} + \Gamma^2 l^2 f(r) d\psi^2 + r^2 d\phi^2 + \frac{r^2}{l^2} dX^2, \quad (5)$$

where $dX^2 = \sum_{i=1}^{n-3} (dx^i)^2$ and Γ is a constant will be fixed later. Note that the coordinates $-\infty < x^i < \infty$ have the dimension of length, while the angular coordinates ψ and ϕ are dimensionless as

usual and range in $[0, 2\pi]$. The motivation for this metric gauge $[g_{tt} \propto -r^2$ and $(g_{rr})^{-1} \propto g_{\psi\psi}]$ instead of the usual Schwarzschild gauge $[(g_{rr})^{-1} \propto g_{tt}]$ comes from the fact that we are looking for a horizonless magnetic solution. The electromagnetic field equation (2) reduces to

$$\beta^2 l^2 \Gamma^2 \left[r F'_{\psi r}(r) + (n-1) F_{\psi r}(r) \right] + (n-1) F_{\psi r}^3(r) = 0, \quad (6)$$

where the prime denotes a derivative with respect to the r coordinate. The solution of Eq. (6) can be written as

$$F_{\psi r} = \frac{2\Gamma^2 l^{n-2} q}{r^{n-1} \sqrt{1-\eta}}, \quad (7)$$

where q is an arbitrary constant and

$$\eta = \frac{4\Gamma^2 q^2 l^{2(n-3)}}{\beta^2 r^{2(n-1)}}. \quad (8)$$

Equation (7) shows that r should be greater than $r_{01} = (2\Gamma q l^{n-3}/\beta)^{1/(n-1)}$ in order to have a real nonlinear electromagnetic field and consequently a real spacetime. To find the function $f(r)$, one may use any components of Eq. (1). The simplest equation is the rr component of these equations which can be written as

$$(n-1)\{l^2 r^{n-4}[2(n-2)(n-3)\alpha f - r^2]f' + (n-2)l^2 r^{n-5}[(n-3)(n-4)\alpha f - r^2]f + nr^{n-1}\} + 4\beta^2 l^2 r^{n-1}(1 - \sqrt{1-\eta}) = 0. \quad (9)$$

The solution of Eq. (9), which also satisfies all the other components of the gravitational field equations (1), can be written as

$$f(r) = \frac{r^2}{2(n-2)(n-3)\alpha} \left(1 - \sqrt{g(r)}\right), \quad (10)$$

where

$$g(r) = 1 + 16 \frac{(n-3)\alpha\beta^2\eta}{n} {}_2F_1 \left(\left[\frac{1}{2}, \frac{n-2}{2n-2} \right], \left[\frac{3n-4}{2n-2} \right], \eta \right) + 4(n-2)(n-3)\alpha \left[\frac{4\beta^2(\sqrt{1-\eta}-1)}{n(n-1)} - \frac{1}{l^2} - \frac{2ml}{r^n} \right], \quad (11)$$

In Eq. (11) m is an integration constant which is related to geometrical mass of the spacetime and ${}_2F_1([a, b], [c], z)$ is the hypergeometric function which may be defined as [17]

$${}_2F_1 \left(\left[\frac{1}{2}, \frac{n-2}{2n-2} \right], \left[\frac{3n-4}{2n-2} \right], bu^{2n-2} \right) = \frac{n-2}{u^{n-2}} \int \frac{u^{n-3}}{\sqrt{1-bu^{2n-2}}} du.$$

One may note that the above asymptotically AdS solution reduces to Einstein-Born-Infeld solution when α vanishes and reduces to the solution introduced in [18] as β goes to infinity. When m and q are zero, the vacuum solution is

$$f(r) = \frac{r^2}{2(n-2)(n-3)\alpha} \left(1 - \sqrt{1 - \frac{4(n-2)(n-3)\alpha}{l^2}} \right). \quad (12)$$

Equation (12) shows that for a positive value of α , this parameter should satisfies $\alpha \leq l^2/4(n-2)(n-3)$. Also note that l_{eff} for the AdS solution of the theory is

$$l_{\text{eff}} = [2(n-2)(n-3)\alpha]^{1/2} \left(1 - \sqrt{1 - \frac{4(n-2)(n-3)\alpha}{l^2}} \right)^{-1/2}, \quad (13)$$

which reduces to l as α goes to zero.

To have a real spacetime, the function $g(r)$ should be positive. This occurs provided the mass parameter $m \leq m_0$, where m_0 is the value of m calculated from the equation $g(r = r_{01}) = 0$ given as:

$$m_0 = \frac{r_{01}^n}{2l} \left\{ \frac{1}{4(n-2)(n-3)\alpha} - \frac{1}{l^2} + \frac{4\beta^2}{n(n-2)} {}_2F_1 \left(\left[\frac{1}{2}, \frac{n-2}{2n-2} \right], \left[\frac{3n-4}{2n-2} \right], 1 \right) - \frac{4\beta^2}{n(n-1)} \right\}$$

In this case the metric function $f(r)$ is real for $r \geq r_0 = r_{01}$. For the case of $m > m_0$, r should be greater than r_0 in order to have a real spacetime, where r_0 is the largest real root of $g(r) = 0$. Figure 1 shows the zeros of $g(r)$ and $1 - \eta$ (r_0 and r_{01} respectively) for $m > m_0$.

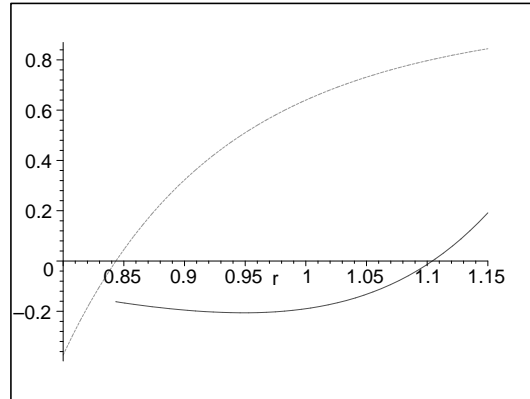


FIG. 1: The functions $g(r)$ (continuous-line) and $1 - \eta$ (dashed-line) versus r for $n = 4$, $q = 0.3$, $m = 0.8$, $l = \Gamma = \beta = 1$, and $\alpha = 0.2$.

In order to study the general structure of this solution, we first look for curvature singularities. It is easy to show that the Kretschmann invariant $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverges at $r = r_0$, it is finite for $r > r_0$ and goes to zero as $r \rightarrow \infty$. Therefore one might think that there is a curvature singularity

located at $r = r_0$. Two cases happen. In the first case the function $f(r)$ has no real root greater than r_0 , and therefore we encounter with a naked singularity which we are not interested in it. So we consider only the second case which the function has one or more real root(s) larger than r_0 . In this case the function $f(r)$ is negative for $r < r_+$, and positive for $r > r_+$ where r_+ is the largest real root of $f(r) = 0$. Indeed, g_{rr} and $g_{\psi\psi}$ are related by $f(r) = g_{rr}^{-1} = \Gamma^{-2} l^{-2} g_{\psi\psi}$, and therefore when g_{rr} becomes negative (which occurs for $r_0 < r < r_+$) so does $g_{\psi\psi}$. This leads to an apparent change of signature of the metric from $(n-1)+$ to $(n-2)+$, and therefore indicates that r should be greater than r_+ . Thus the coordinate r assumes the value $r_+ \leq r < \infty$. The function $f(r)$ given in Eq. (10) is positive in the whole spacetime and is zero at $r = r_+$. The Kretschmann scalar is a linear combination of the square of f'' , f'/r and f/r^2 . Since these terms do not diverge in the range $r_+ \leq r < \infty$, one finds that the Kretschmann scalar is finite. Also one can show that other curvature invariants (such as Ricci scalar, Ricci square, Weyl square and so on) are functions of f'' , f'/r and f/r^2 and therefore the spacetime has no curvature singularity in the range $r_+ \leq r < \infty$. In order to avoid conic singularity at $r = r_+$ in the $(r - \psi)$ -section, one may fix the factor $\Gamma = 1/[lf'(r_+)]$ in the metric.

Now, we investigate the wormhole interpretation of the above solution. It satisfies the so-called flare-out condition when $r = r_+$. This can be seen by embedding the 2-surface of constant t , ψ and x^i 's with the metric

$$ds^2 = \frac{dr^2}{f(r)} + r^2 d\phi^2, \quad (14)$$

in an (unphysical) three-dimensional Euclidean flat space, which has the metric

$$ds^2 = dr^2 + r^2 d\phi^2 + dz^2 \quad (15)$$

in cylindrical coordinates. The surface described by the function $z = z(r)$ satisfies

$$\frac{dr}{dz} = \sqrt{\frac{f(r)}{1-f(r)}} = 0, \quad (16)$$

$$\frac{d^2r}{dz^2} = \frac{f'}{2[1-f]^2} > 0, \quad (17)$$

which show that it has the characteristic shape of a wormhole, as illustrated in Figs. 1 and 2 of Ref. [13]. Next we consider the Weak Energy Condition (WEC) for the above solution. Using the orthonormal contravariant (hatted) basis vectors

$$\mathbf{e}_{\hat{t}} = \frac{l}{r} \frac{\partial}{\partial t}, \quad \mathbf{e}_{\hat{r}} = f^{1/2} \frac{\partial}{\partial r}, \quad \mathbf{e}_{\hat{\psi}} = \frac{1}{\Gamma l f^{1/2}} \frac{\partial}{\partial \psi}, \quad \mathbf{e}_{\hat{\phi}} = r^{-1} \frac{\partial}{\partial \phi}, \quad \mathbf{e}_{\hat{x}^i} = \frac{l}{r} \frac{\partial}{\partial x^i},$$

the mathematics and physical interpretations become simplified. It is a matter of straight forward calculations to show that the components of stress-energy tensor are

$$\begin{aligned} T_{\hat{t}\hat{t}} &= -T_{\hat{\phi}\hat{\phi}} = -T_{\hat{i}\hat{i}} = \frac{\beta^2}{4\pi} \left(\left[1 + \left(\frac{F_{\psi r}}{\Gamma l \beta} \right)^2 \right]^{1/2} - 1 \right), \\ T_{\hat{r}\hat{r}} &= T_{\hat{\psi}\hat{\psi}} = \frac{\beta^2}{4\pi} \left(1 - \left[1 + \left(\frac{F_{\psi r}}{\Gamma l \beta} \right)^2 \right]^{-1/2} \right), \end{aligned} \quad (18)$$

which satisfy the weak energy conditions:

$$T_{\hat{t}\hat{t}} \geq 0, \quad T_{\hat{t}\hat{t}} + T_{\hat{i}\hat{i}} = T_{\hat{t}\hat{t}} + T_{\hat{\phi}\hat{\phi}} \geq 0, \quad T_{\hat{t}\hat{t}} + T_{\hat{r}\hat{r}} = T_{\hat{t}\hat{t}} + T_{\hat{\psi}\hat{\psi}} \geq 0. \quad (19)$$

III. ROTATING WORMHOLE SOLUTIONS

First, we want to endow our spacetime solution (5) with a global rotation. In order to add angular momentum to the spacetime, we perform the following rotation boost in the $t - \psi$ plane

$$t \mapsto \Xi t - a\psi, \quad \psi \mapsto \Xi\psi - \frac{a}{l^2}t, \quad (20)$$

where a is a rotation parameter and $\Xi = \sqrt{1 + a^2/l^2}$. Substituting Eq. (20) into Eq. (5) we obtain

$$ds^2 = -\frac{r^2}{l^2} (\Xi dt - a d\psi)^2 + \frac{dr^2}{f(r)} + \Gamma^2 l^2 f(r) \left(\frac{a}{l^2} dt - \Xi d\psi \right)^2 + r^2 d\phi^2 + \frac{r^2}{l^2} dX^2, \quad (21)$$

where $f(r)$ is the same as $f(r)$ given in Eq. (10). The non-vanishing components of electromagnetic field tensor are now given by

$$F_{tr} = \frac{a}{\Xi l^2} F_{r\psi} = \frac{2\Gamma^2 l^{n-4} q a}{r^{n-1} \sqrt{1 - \eta}}. \quad (22)$$

Because of the periodic nature of ψ , the transformation (20) is not a proper coordinate transformation on the entire manifold. Therefore, the metrics (5) and (21) can be locally mapped into each other but not globally, and so they are distinct [19]. Note that this spacetime has no horizon and curvature singularity. One should note that these solutions are different from those discussed in [11], which were electrically charged rotating black brane solutions in Gauss-Bonnet gravity. The electric solutions have black holes, while the magnetic solution interpret as wormhole. It is worthwhile to mention that this solution reduces to the solution of Einstein-Maxwell equation introduced in [18] as α goes to zero.

Second, we generalize the above solution to the case of rotating solution with more rotation parameters. The rotation group in $n + 1$ dimensions is $SO(n)$ and therefore the number of independent rotation parameters is $[n/2]$, where $[x]$ is the integer part of x . The generalized solution

with $k \leq [n/2]$ rotation parameters can be written as

$$ds^2 = -\frac{r^2}{l^2} \left(\Xi dt - \sum_{i=1}^k a_i d\psi^i \right)^2 + \Gamma^2 f(r) \left(\sqrt{\Xi^2 - 1} dt - \frac{\Xi}{\sqrt{\Xi^2 - 1}} \sum_{i=1}^k a_i d\psi^i \right)^2 + \frac{dr^2}{f(r)} + \frac{r^2}{l^2(\Xi^2 - 1)} \sum_{i < j}^k (a_i d\psi_j - a_j d\psi_i)^2 + r^2 d\phi^2 + \frac{r^2}{l^2} dX^2, \quad (23)$$

where $\Xi = \sqrt{1 + \sum_i^k a_i^2/l^2}$, dX^2 is the Euclidean metric on the $(n-k-2)$ -dimensional submanifold and $f(r)$ is the same as $f(r)$ given in Eq. (10). The non-vanishing components of electromagnetic field tensor are

$$F_{tr} = \frac{(\Xi^2 - 1)}{\Xi a_i} F_{r\psi^i} = \frac{2ql^{n-3}\Gamma^2\sqrt{\Xi^2 - 1}}{r^{n-1}\sqrt{1-\eta}}. \quad (24)$$

A. Conserved Quantities

In general the conserved quantities are divergent when evaluated on the solutions. A systematic method of dealing with this divergence for asymptotically AdS solutions of Einstein gravity is through the use of the counterterms method inspired by the anti-de Sitter conformal field theory (AdS/CFT) correspondence [20]. For asymptotically AdS solutions of Lovelock gravity with flat boundary, $\hat{R}_{abcd}(\gamma) = 0$, the finite energy momentum tensor is [21, 22]

$$T^{ab} = \frac{1}{8\pi} \{ (K^{ab} - K\gamma^{ab}) + 2\alpha(3J^{ab} - J\gamma^{ab}) - \left(\frac{n-1}{L} \right) \gamma^{ab} \}, \quad (25)$$

where L is

$$L = \frac{3\sqrt{\chi}(1 - \sqrt{1-\chi})^{3/2}}{\sqrt{8}[1 - (1 + \frac{\chi}{2})\sqrt{1-\chi}]} l, \quad (26)$$

and $\chi = 4(n-2)(n-3)\alpha/l^2$. In Eq. (25), K^{ab} is the extrinsic curvature of the boundary, K is its trace, γ^{ab} is the induced metric of the boundary, and J is trace of J^{ab}

$$J_{ab} = \frac{1}{3} (K_{cd}K^{cd}K_{ab} + 2KK_{ac}K_b^c - 2K_{ac}K^{cd}K_{db} - K^2K_{ab}). \quad (27)$$

One may note that when α goes to zero, the finite stress-energy tensor (25) reduces to that of asymptotically AdS solutions of Einstein gravity with flat boundary.

To compute the conserved charges of the spacetime, we choose a spacelike surface \mathcal{B} in $\partial\mathcal{M}$ with metric σ_{ij} , and write the boundary metric in ADM (Arnowitt-Deser-Misner) form:

$$\gamma_{ab}dx^a dx^a = -N^2 dt^2 + \sigma_{ij} (d\phi^i + V^i dt) (d\phi^j + V^j dt), \quad (28)$$

where the coordinates ϕ^i are the angular variables parameterizing the hypersurface of constant r around the origin, and N and V^i are the lapse and shift functions respectively. When there is a Killing vector field ξ on the boundary, then the quasilocal conserved quantities associated with the stress tensors of Eq. (25) can be written as

$$\mathcal{Q}(\xi) = \int_{\mathcal{B}} d^{n-1} \varphi \sqrt{\sigma} T_{ab} n^a \xi^b, \quad (29)$$

where σ is the determinant of the metric σ_{ij} , and n^a is the timelike unit normal vector to the boundary \mathcal{B} . In the context of counterterm method, the limit in which the boundary \mathcal{B} becomes infinite (\mathcal{B}_∞) is taken, and the counterterm prescription ensures that the action and conserved charges are finite. No embedding of the surface \mathcal{B} in to a reference of spacetime is required and the quantities which are computed are intrinsic to the spacetimes.

For our case, the magnetic solutions of Gauss-Bonnet gravity, the first Killing vector is $\xi = \partial/\partial t$, therefore its associated conserved charge is the total mass of the wormhole per unit volume V_{n-k-2} , given by

$$M = \int_{\mathcal{B}} d^{n-1} x \sqrt{\sigma} T_{ab} n^a \xi^b = \frac{(2\pi)^k}{4} [n(\Xi^2 - 1) + 1] \Gamma m. \quad (30)$$

For the rotating solutions, the conserved quantities associated with the rotational Killing symmetries generated by $\zeta_i = \partial/\partial \phi^i$ are the components of angular momentum per unit volume V_{n-k-2} calculated as

$$J_i = \int_{\mathcal{B}} d^{n-1} x \sqrt{\sigma} T_{ab} n^a \zeta_i^b = \frac{(2\pi)^k}{4} \Gamma n \Xi m a_i. \quad (31)$$

Next, we calculate the electric charge of the solutions. To determine the electric field we should consider the projections of the electromagnetic field tensor on special hypersurfaces. The normal to such hypersurfaces for the spacetimes with a longitudinal magnetic field is

$$u^0 = \frac{1}{N}, \quad u^r = 0, \quad u^i = -\frac{N^i}{N},$$

and the electric field is $E^\mu = g^{\mu\rho} F_{\rho\nu} u^\nu$. Then the electric charge per unit volume V_{n-k-2} can be found by calculating the flux of the electromagnetic field at infinity, yielding

$$Q = \frac{(2\pi)^k}{2} \Gamma q \sqrt{\Xi^2 - 1}. \quad (32)$$

Note that the electric charge is proportional to the magnitude of rotation parameters and is zero for the static solutions.

IV. CLOSING REMARKS

Considering both the nonlinear invariant terms constructed by electromagnetic field tensor ($F_{\mu\nu}F^{\mu\nu}$) and quadratic invariant terms constructed by Riemann tensor (R^2 , $R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$) in action, we have obtained a new class of rotating spacetimes in various dimensions, with negative cosmological constant. These solutions are asymptotically anti-de Sitter and reduce to the solutions of Gauss-Bonnet-Maxwell gravity when $\beta \rightarrow \infty$ and reduce to those of Einstein-Born-Infeld gravity as $\alpha \rightarrow 0$. This class of solutions yields an $(n+1)$ -dimensional spacetime with a longitudinal nonlinear and nonsingular magnetic field (the only nonzero component of the vector potential is $A_\phi(r)$) generated by a static magnetic source. We have found that these solutions have no curvature singularity and no horizons and may be interpreted as a traversable wormhole near $r = r_+$. Also, we found that the weak energy condition is not violated at the throat, which shows there is no exotic matter near the throat.

Also we have generalized these solutions to the case of rotating spacetimes with a longitudinal magnetic field. For the rotating wormhole, when a rotational parameter is nonzero, the wormhole has a net electric charge density which is proportional to the magnitude of the rotational parameter given by $\sqrt{\Xi^2 - 1}$. For static case, the electric field vanishes, and therefore the wormhole has no net electric charge. Finally, we applied the counterterm method and calculated the conserved quantities of the solutions. We have found that the conserved quantities do not depend on the Born-Infeld parameter β .

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